Pearson Edexcel

Mark Scheme (Results)

January 2020

Pearson Edexcel International GCSE
In Further Pure Mathematics (4PM1)
Paper 02R

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme.
  Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

- **Types of mark**
  - M marks: method marks
  - A marks: accuracy marks – can only be awarded when relevant M marks have been gained
  - B marks: unconditional accuracy marks (independent of M marks)

- **Abbreviations**
  - cao – correct answer only
  - cso – correct solution only
  - ft – follow through
  - isw – ignore subsequent working
  - SC - special case
  - oe – or equivalent (and appropriate)
  - dep – dependent
  - indep – independent
  - awrt – answer which rounds to
  - eeoo – each error or omission

- **No working**
  If no working is shown then correct answers may score full marks
  If no working is shown then incorrect (even though nearly correct) answers score no marks.

- **With working**

  If it is clear from the working that the "correct" answer has been obtained from incorrect working, award 0 marks.

  If a candidate misreads a number from the question: eg. uses 252 instead of 255; follow through their working and deduct 2A marks from any gained provided the work has not been simplified. (Do not deduct any M marks gained.)

  If there is a choice of methods shown, then award the lowest mark, unless the subsequent working makes clear the method that has been used

  Examiners should send any instance of a suspected misread to review (but see above for simple misreads).

- **Ignoring subsequent work**

  It is appropriate to ignore subsequent work when the additional work does not change the answer in a way that is inappropriate for the question: eg. incorrect cancelling of a fraction that would otherwise be correct.

  It is not appropriate to ignore subsequent work when the additional work essentially makes the answer incorrect eg algebra.

- **Parts of questions**

  Unless allowed by the mark scheme, the marks allocated to one part of the question CANNOT be awarded to another.

General Principles for Further Pure Mathematics Marking

(but note that specific mark schemes may sometimes override these general principles)

Method mark for solving a 3 term quadratic equation:

1. Factorisation:

$$\left(x^2+bx+c\right)=\left(x+p\right)\left(x+q\right), \text{ where } |pq|=|c| \text{ leading to } x=\ldots.$$

$$\left(ax^2+bx+c\right)=\left(mx+p\right)\left(nx+q\right) \text{ where } |pq|=|c| \text{ and } |mn|=|a| \text{ leading to } x=\ldots.$$

2. Formula:

Attempt to use the **correct** formula (shown explicitly or implied by working) with values for *a*, *b* and *c*, leading to $x=\ldots.$

3. Completing the square:

$$x^2+bx+c=0:\ \left(x\pm\tfrac{b}{2}\right)^2\pm q\pm c=0,\quad q\neq 0 \qquad \text{leading to } x=\ldots.$$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. $\left(x^n \to x^{n-1}\right)$

2. Integration:

Power of at least one term increased by 1. $\left(x^n \to x^{n+1}\right)$

Use of a formula:

Generally, the method mark is gained by **either**

quoting a correct formula and attempting to use it, even if there are mistakes in the substitution of values

or, where the formula is not quoted, the method mark can be gained by implication from the substitution of correct values and then proceeding to a solution.

Answers without working:

The rubric states "Without sufficient working, correct answers may be awarded no marks".

General policy is that if it could be done "in your head" detailed working would not be required. (Mark schemes may override this eg in a case of "prove or show...."

Exact answers:

When a question demands an exact answer, all the working must also be exact. Once a candidate loses exactness by resorting to decimals the exactness cannot be regained.

Rounding answers (where accuracy is specified in the question)

Penalise only once per question for failing to round as instructed - ie giving more digits in the answers. Answers with fewer digits are automatically incorrect, but the is rule may allow the mark to be awarded before the final answer is given.
before the final answer is given.

MARK SCHEME

Question number	Scheme	Marks
1 (a)	$l = r\theta \Rightarrow l = 13\times 2 = 26$ (cm)	B1 [1]
(b)	$A = \frac{\theta}{2}r^2 = \frac{2}{2}\times 13^2 = 169$ (cm^2)	M1A1 [2]
		Total 3 marks
(a) **B1** **(b)** **M1** **A1**	 $l = 26$ Use of $A = \frac{\theta}{2}r^2$ or $A = \frac{1}{2}rl$ or $A = \frac{l^2}{2\theta}$ $A = 169$	

Question number	Scheme	Marks
2 (a)	Line l_1 $\quad m = \frac{-8}{4} = -2 \quad y - 8 = -2(x) \Rightarrow y + 2x = 8$ Line l_2 $\quad m = \frac{-4}{6} = -\frac{2}{3} \quad y - 4 = -\frac{2}{3}(x) \Rightarrow 3y + 2x = 12$	M1A1 A1 [3]
(b)	$x \ge 0 \quad y + 2x \le 8 \quad 3y + 2x \ge 12$ Accept < and >	B1B1ftB1ft [3]
		Total 6 marks
(a) **M1** **A1** **A1** **(b)** **B1** **B1ft** **B1ft**	 Calculating the gradient of either l_1 or l_2 $y + 2x = 8$ $3y + 2x = 12$ **NB** If both are correct but not in the form $ax + by = c$ then award A1A0 For all 3 marks accept < and > instead of $\le$ and $\ge$ $x \ge 0$ $y + 2x \le 8$ oe (ft l_1) $3y + 2x \ge 12$ oe (ft l_2)	

Question number	Scheme	Marks
3	$33=\frac{1}{2}\times 11\times 12\times \sin\angle ABC \Rightarrow \sin\angle ABC=\frac{1}{2}$ $\left[30^{\circ} \text{ or } 150^{\circ}\right]$	M1A1
	$AC=\sqrt{11^2+12^2-2\times 11\times 12\cos 30^{\circ}}=6.0306...\approx 6.03$ (cm) $AC=\sqrt{11^2+12^2-2\times 11\times 12\cos 150^{\circ}}=22.2178...\approx 22.2$ (cm)	M1A1 A1 [5]
		Total 5 marks
M1	Use of $\frac{1}{2}ab\sin C=33$ (must be set = 33)	
A1	30° and 150° (Accept $\frac{\pi}{6}$ and $\frac{5\pi}{6}$)	
M1 **A1** **A1**	Use of $c^2=a^2+b\ \ -2ab\cos C$ (square root not required for M1) 6.03 22.2 **NB** If both answers are not given to 3sf but are correct then award A1A0	

Question number	**Scheme**	**Marks**
4 (a)	$A=\frac{1}{2}\times 8\times 8\times \sin 60^{\circ}=\left(16\sqrt{3}\right)$	M1
	$48\sqrt{3}=\frac{1}{3}\times '16\sqrt{3}'\times h \Rightarrow h=9$ *	M1 A1cso [3]
(b)	$BX=\sqrt{9^2+8^2}=\sqrt{145}$	M1
	$\angle BXC=\frac{'145'+'145'-8^2}{2\times '\sqrt{145}'\times '\sqrt{145}'}=38.8025...^{\circ}\approx 38.8^{\circ}$	M1A1 [3]
(c)	Let midpoint of BC be M $AM=\sqrt{8^2-4^2}=\left(4\sqrt{3}\right)$ or $MX=\sqrt{145-4^2}=\left(\sqrt{129}\right)$	M1
	$\angle XMA=\tan^{-1}\left(\frac{9}{'4\sqrt{3}'}\right)$ or $\angle XMA=\sin^{-1}\left(\frac{9}{'\sqrt{129}'}\right)$ or $\angle XMA=\cos^{-1}\left(\frac{'4\sqrt{3}'}{'\sqrt{129}'}\right)$ $=52.4109...^{\circ}\approx 52.4^{\circ}$	M1 A1 [3]
	ALT 1	
	Let midpoint of BC be M $\left(XA\right)^2=\left(AM\right)^2+\left(XM\right)^2-2(AM)(XM)\cos\theta$	{M1}
	$9^2=\left(8^2-4^2\right)+\left(8^2+9^2-4^2\right)-2\sqrt{8^2-4^2}\sqrt{8^2+9^2-4^2}\cos\theta$ $\theta=52.4109...^{\circ}\approx 52.4^{\circ}$	{M1} {A1} [3]
		Total 9 marks

(a)	
M1	Use of $\frac{1}{2}ab\sin C$ (may be implied by $\left(16\sqrt{3}\right)$
M1	Use of $\frac{1}{3}\times\text{'Area of base'}\times h$
A1 cso	Obtains the **given** answer with no errors in the working
(b)	
M1	Use of $\sqrt{(\text{part } a)^2+8^2}$ (may be implied by $\sqrt{145}$)
M1	Use the cosine rule, either form. If not for angle BXC there must be a complete method shown for obtaining BXC (follow through their BX)
A1	awrt 38.8°
(c)	
M1	Use of Pythagoras' to find the length of AM or MX (may be implied by $4\sqrt{3}$ or $\sqrt{129}$)
M1	$\tan^{-1}\left(\frac{9}{'4\sqrt{3}'}\right)$ or $\sin^{-1}\left(\frac{9}{'\sqrt{129}'}\right)$ or $\cos^{-1}\left(\frac{'4\sqrt{3}'}{'\sqrt{129}'}\right)$
A1	awrt 52.4°

ALT	
M1	Use of cosine rule using *AX*, *AM* and *XM* e.g. $(XA)^2 = (AM)^2 + (XM)^2 - 2(AM)(XM)\cos\theta$
M1	Correct values substituted into the cosine rule in any form e.g. $9^2 = (8^2 - 4^2) + (8^2 + 9^2 - 4^2) - 2\sqrt{8^2 - 4^2}\sqrt{8^2 + 9^2 - 4^2}\cos\theta$
A1	awrt 52.4°

Question number	Scheme	Marks
5 (a)	$(\alpha+\beta)^3=\alpha^3+3\alpha^2\beta+3\alpha\beta^2+\beta^3 \Rightarrow \alpha^3+\beta^3$ $=(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)$*	M1A1 cso [2]
(b)	$\alpha+\beta=-\frac{3}{2}\qquad \alpha\beta=\frac{6}{2}=3$ $\left[\alpha^3+\beta^3=(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)\right]$ $\alpha^3+\beta^3=\left(-\frac{3}{2}\right)^3-3\times3\times\left(-\frac{3}{2}\right)=\frac{81}{8}$	B1 B1 [2]
(c)	$(\alpha^2+\beta^2)^2=\alpha^4+2\alpha^2\beta^2+\beta^4 \Rightarrow \alpha^4+\beta^4=(\alpha^2+\beta^2)^2-2(\alpha\beta)^2$ *	M1A1 cso [2]
(d)	$\alpha^2+\beta^2=\left(-\frac{3}{2}\right)^2-2\times3=-\frac{15}{4}$ Sum $\quad(\alpha^3-\beta)+(\beta^3-\alpha)=\alpha^3+\beta^3-(\alpha+\beta)=\frac{81}{8}-\left(-\frac{3}{2}\right)=\frac{93}{8}$ Product $\quad(\alpha^3-\beta)\times(\beta^3-\alpha)=(\alpha\beta)^3-(\alpha^4+\beta^4)+\alpha\beta$ $=27-\left[\left(-\frac{15}{4}\right)^2-2\times3^2\right]+3=\frac{543}{16}$ Equation $\quad x^2-\frac{93}{8}x+\frac{543}{16}=0\Rightarrow 16x^2-186x+543=0$	B1 B1 M1 A1 M1A1 [6]
		Total 12 marks

(a)	
M1	$(\alpha+\beta)^3=\alpha^3+3\alpha^2\beta+3\alpha\beta^2+\beta^3$
A1 cso	Obtains the **given** answer with no errors in the working
(b)	
B1	$\alpha+\beta=-\frac{3}{2}$ and $\alpha\beta=\frac{6}{2}=3$
B1	$\alpha^3+\beta^3=\frac{81}{8}$
(c)	
M1	$(\alpha^2+\beta^2)^2=\alpha^4+2\alpha^2\beta^2+\beta^4$
A1 cso	Obtains the **given** answer with no errors in the working

(d)	
B1	$\alpha^2+\beta^2=-\frac{15}{4}$ $\left(\text{May be implied by } \alpha^4+\beta^4=-\frac{63}{16}\right)$
B1	$\left(\alpha^3-\beta\right)+\left(\beta^3-\alpha\right)=\frac{93}{8}$
M1	$\left(\alpha^3-\beta\right)\times\left(\beta^3-\alpha\right)=(\alpha\beta)^3-\left(\alpha^4+\beta^4\right)+\alpha\beta$
A1	$\left(\alpha^3-\beta\right)\times\left(\beta^3-\alpha\right)=\frac{543}{16}$
M1	$x^2-\text{'sum'}x+\text{'product'}(=0)$
A1	$16x^2-186x+543=0$

Question number	Scheme	Marks
6 (a)	$\left[V=\frac{1}{3}\pi r^2 h\right]$	
	$\tan 30^\circ = \frac{r}{h} \Rightarrow r = \frac{h}{\sqrt{3}}$	M1
	$V=\frac{1}{3}\pi\left(\frac{h}{\sqrt{3}}\right)^2 h \Rightarrow V=\frac{1}{9}\pi h^3$ *	A1 cso [2]
(b)	$\frac{dV}{dh}=\frac{\pi h^2}{3}$	M1
	$\frac{dh}{dt}=\frac{dh}{dV}\times\frac{dV}{dt} \Rightarrow \frac{dh}{dt}=\frac{3}{\pi h^2}\times -0.9=\frac{3}{\pi\times 1.2^2}\times -0.9=-0.59683...$	M1M1
	≈ -0.597 cm/s [Accept an answer of ± 0.597 cm/s]	A1 [4]
		Total 6 marks

(a)	
M1	Finding $r=\frac{h}{\sqrt{3}}$ and substituting into $V=\frac{1}{3}\pi r^2 h$ (Allow $r=h\tan 30^\circ$)
A1 cso	Obtains the **given** answer with no errors in the working
(b)	
M1	$\frac{dV}{dh}=\frac{\pi h^2}{3}$
M1	$\frac{dh}{dt}=\frac{dh}{dV}\times\frac{dV}{dt}$
M1	$\frac{dh}{dt}=\frac{3}{\pi h^2}\times \pm 0.9$
A1	± 0.597 (cm/s)

Question number	Scheme	Marks
7 (a)	$\dfrac{ar^6}{ar^3}=r^3=\dfrac{e^{\frac{2x+1}{2}}}{e^{x+2}}=\dfrac{e^x\times e^{\frac{1}{2}}}{e^x\times e^2}=e^{-\frac{3}{2}}\Rightarrow r=e^{-\frac{1}{2}}$ * **ALT** $\dfrac{e^{x+2}}{r^3}r^6=e^{\frac{2x+1}{2}}\Rightarrow r^3=e^{x+\frac{1}{2}-x+2}\Rightarrow r^3=e^{-\frac{3}{2}}\Rightarrow r=e^{-\frac{1}{2}}$ *	M1M1A1 cso [3] {M1} {M1}{A1} cso [3]
(b)	$ar^3=ae^{-\frac{3}{2}}=e^{x+2}\Rightarrow a=\dfrac{e^{x+2}}{e^{-\frac{3}{2}}}=\dfrac{e^x\times e^2}{e^{-\frac{3}{2}}}=e^{x+\frac{7}{2}}$ oe **ALT** $a=\dfrac{e^{x+2}}{e^{-\frac{3}{2}}}=e^{x+2+\frac{3}{2}}=e^{x+\frac{7}{2}}$ oe	M1M1A1 [3] {M1}{M1} {A1} [3]
(a)		
M1	Use of $\dfrac{ar^6}{ar^3}=r^3$	
M1	Using $e^{a+b}=e^a\times e^b$ to simplify leading to $r^3=e^c$ where c is a number	
A1 cso	Obtains the **given** answer with no errors in the working	
ALT		
M1	For rearranging to make a subject and substituting into the other equation	
M1	Using $e^{a-b}=e^a\div e^b$ to simplify leading to $r^3=e^c$ where c is a number	
A1 cso	Obtains the **given** answer with no errors in the working	
(b)		
M1	For $a=\dfrac{e^{x+2}}{e^{-\frac{3}{2}}}$	
M1	Using $e^{a+b}=e^a\times e^b$ to simplify to $a=\dfrac{e^x\times e^2}{e^{-\frac{3}{2}}}$	
A1	$e^{x+\frac{7}{2}}$ oe	
ALT		
M1	For $a=\dfrac{e^{x+2}}{e^{-\frac{3}{2}}}$	
M1	Using $e^{a+b}=e^a\times e^b$ to simplify to $e^{x+2+\frac{3}{2}}$	
A1	$e^{x+\frac{7}{2}}$ oe	

(c)	$S_\infty = \dfrac{e^{x+\frac{7}{2}}}{1-e^{-\frac{1}{2}}} = \dfrac{e^{x+\frac{7}{2}}}{\frac{e^{\frac{1}{2}}-1}{e^{\frac{1}{2}}}} = \dfrac{e^{x+4}}{e^{\frac{1}{2}}-1} \Rightarrow p = x+4$	M1M1A1 [3]
	ALT 1	
	$S_\infty = \dfrac{e^{x+\frac{7}{2}}}{1-e^{-\frac{1}{2}}} = \dfrac{e^{x+\frac{7}{2}}}{1-e^{-\frac{1}{2}}} \times \dfrac{e^{\frac{1}{2}}}{e^{\frac{1}{2}}} = \dfrac{e^{x+4}}{e^{\frac{1}{2}}-1} \Rightarrow p = x+4$	{M1}{M1} {A1} [3]
	ALT 2	
	$S_\infty = \dfrac{e^{x+\frac{7}{2}}}{1-e^{-\frac{1}{2}}} = \dfrac{e^{p}}{e^{\frac{1}{2}}-1} \Rightarrow e^{x+\frac{7}{2}}\left(e^{\frac{1}{2}}-1\right) = e^{p}\left(1-e^{-\frac{1}{2}}\right)$ $\Rightarrow e^{x+4} - e^{x+\frac{7}{2}} = e^{p} - e^{p-\frac{1}{2}} \Rightarrow p = x+4$	{M1}{M1} {A1} [3]

(c)	
M1	Use of $S_\infty = \dfrac{a}{1-r}$
M1	$1-e^{-\frac{1}{2}} = \dfrac{e^{\frac{1}{2}}-1}{e^{\frac{1}{2}}}$
A1	$p = x+4$
ALT 1	
M1	Use of $S_\infty = \dfrac{a}{1-r}$
M1	Multiplying S_∞ by $\dfrac{e^{\frac{1}{2}}}{e^{\frac{1}{2}}}$
A1	$p = x+4$
ALT 2	
M1	Use of $S_\infty = \dfrac{a}{1-r}$
M1	For simplifying to $e^{x+4} - e^{x+\frac{7}{2}} = e^{p} - e^{p-\frac{1}{2}}$ oe
A1	$p = x+4$

(d)	$e^{x+\frac{7}{2}} \times \left(e^{-\frac{1}{2}}\right)^{17} > 1.6 \Rightarrow e^x \times e^{\left(\frac{7}{2}-\frac{17}{2}\right)} > 1.6 \Rightarrow e^x > \frac{1.6}{e^{-5}}$ $\Rightarrow e^x > 237.46105....$ $\Rightarrow x > \ln(237.46105) \Rightarrow x > 5.4700...$ $\Rightarrow x = 6$ **ALT** $e^{x+\frac{7}{2}} \times \left(e^{-\frac{1}{2}}\right)^{17} > 1.6 \Rightarrow e^{x-5} > 1.6$ $\Rightarrow x-5 > \ln(1.6)$ $\Rightarrow x > \ln(1.6)+5 \Rightarrow x > 5.4700...$ $\Rightarrow x = 6$	M1 M1 M1 A1 [4] {M1} {M1} {M1} {A1} [4]
		Total 13 marks

(d)	**NB** Allow use of = for M marks
M1	Use of $ar^{17} > 1.6$
M1	Simplifying to $e^x > \ldots$
M1	Using logs to reach $x > \ln \ldots$
A1	$x = 6$
ALT	
M1	Use of $ar^{17} > 1.6$
M1	Simplifying to $e^{x-5} > \ldots$ ft their a
M1	Using logs to reach $x > \ln \ldots + 5$ ft their a
A1	$x = 6$

Question number	Scheme	Marks
8 (a)	$k = 2$	B1 [1]
(b)	$2+3\cos A-\sin A-3\sin 2A-2\cos^2 A=3\cos A-\sin A-6\sin A\cos A+2\sin^2 A$ $3\cos A-\sin A-6\sin A\cos A+2\sin^2 A=3\cos A(1-2\sin A)-\sin A(1-2\sin A)$ $3\cos A(1-2\sin A)-\sin A(1-2\sin A)=(1-2\sin A)(3\cos A-\sin A)$ $\Rightarrow p=3,\ q=1,\ r=2$	B1 M1 A1 [3]
	ALT	
	$2+3\cos A-\sin A-3\sin 2A-2\cos^2 A=3\cos A-\sin A-6\sin A\cos A+2\sin^2 A$ $(p\cos A-\sin A)(q-r\sin A)=pq\cos A-q\sin A-pr\cos A\sin A+r\sin^2 A$ $pq\cos A-q\sin A-pr\cos A\sin A+r\sin^2 A$ $\Rightarrow p=3,\ q=1,\ r=2$	{B1} {M1} {A1} [3]
(c)	$(1-2\sin 2\theta)(3\cos 2\theta-\sin 2\theta)=0$ $\Rightarrow 3\cos 2\theta-\sin 2\theta=0\Rightarrow \tan 2\theta=3$ $\Rightarrow 1-2\sin 2\theta=0\Rightarrow \sin 2\theta=\frac{1}{2}$ $\tan 2\theta=3\Rightarrow 2\theta=1.2490...,\ 4.3906...\ \Rightarrow \theta=0.625,\ 2.20$ $\sin 2\theta=\frac{1}{2}\Rightarrow 2\theta=0.5235...,\ 2.6179...\ \Rightarrow \theta=0.262,\ 1.31\ \left[\frac{\pi}{12},\frac{5\pi}{12}\right]$	 B1 B1 M1A1 M1A1 [6]
		Total 10 marks
(a) **B1**	$k = 2$	
(b) **B1**	Substituting $k = 2$ and use of $\sin^2 A+\cos^2 A=1$ to obtain $3\cos A-\sin A-6\sin A\cos A+2\sin^2 A$	
M1	Factorising to obtain $(1-2\sin A)(3\cos A-\sin A)$	
A1	$p=3,\ q=1,\ r=2$ If p, q and r are stated then they must be correct (may be implied by a correct factorisation if p, q and r are not stated)	
ALT		
B1	Substituting $k = 2$ and use of $\sin^2 A+\cos^2 A=1$ to obtain $3\cos A-\sin A-6\sin A\cos A+2\sin^2 A$	
M1	Expanding $(p\cos A-\sin A)(q-r\sin A)$ to obtain $pq\cos A-q\sin A-pr\cos A\sin A+r\sin^2 A$	
A1	$p=3,\ q=1,\ r=2$	

(c)	
B1	$\tan 2\theta = 3$
B1	$\sin 2\theta = \frac{1}{2}$
M1	$2\theta = 1.24(90...,)\ 4.39(06...\ )$
A1	$\theta = 0.625,\ 2.2(0)$
M1	$2\theta = 0.5235...,\ 2.61(79...)$ allow $\left[\frac{\pi}{6}, \frac{5\pi}{6}\right]$
A1	$\theta = 0.262,\ 1.31$ allow $\left[\frac{\pi}{12}, \frac{5\pi}{12}\right]$

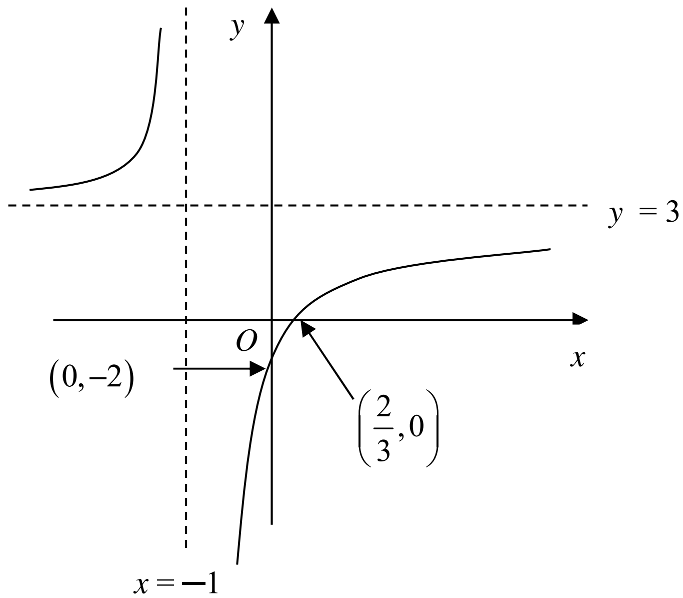
Question number	Scheme	Marks
9 (a)	$\frac{1}{(2-x)^3}=(2-x)^{-3}=\frac{1}{8}\left(1-\frac{x}{2}\right)^{-3}\Rightarrow p=\frac{1}{8},\ q=\frac{1}{2}$	B1B1 [2]
(b)	$\frac{1}{8}\left(1-\frac{x}{2}\right)^{-3}=\frac{1}{8}\left[1+(-3)\left(-\frac{x}{2}\right)+\frac{(-3)(-4)\left(-\frac{x}{2}\right)^2}{2!}+\frac{(-3)(-4)(-5)\left(-\frac{x}{2}\right)^3}{3!}\right]$	M1
	$=\frac{1}{8}+\frac{3}{16}x+\frac{3}{16}x^2+\frac{5}{32}x^3+\ldots$	A1A1 [3]
(c)	$(a+bx)\left(\frac{1}{8}+\frac{3}{16}x+\frac{3}{16}x^2\right)=\frac{a}{8}+x\left(\frac{2b}{16}+\frac{3a}{16}\right)+\left\{x^2\left(\frac{3a}{16}+\frac{3b}{16}\right)\right\}$	M1
	$\Rightarrow\frac{3}{8}=\frac{a}{8}\Rightarrow a=3$	A1
	$\Rightarrow-\frac{43}{16}=\frac{2b+3a}{16}\Rightarrow 2b=-43-9=-52\Rightarrow b=-26$	A1
		[3]
(d)	$\frac{3a+3b}{16}=\frac{9-78}{16}=-\frac{69}{16}$ oe	M1A1 [2]
		Total 10 marks

(a)	**NB** If p and q are stated then they must be correct but if p and q are not stated then $\frac{1}{8}\left(1-\frac{x}{2}\right)^{-3}$ scores B1B1
B1	$p=\frac{1}{8}$ (Allow $p=2^{-3}$)
B1	$q=\frac{1}{2}$

(b)	
M1	Attempts to use the binomial expansion for their $(1-qx)^{-3}$. Must have first term 1, three more terms with ascending powers of *x*, 2 or 2! and 6 or 3! seen , and their $\left(-\frac{x}{2}\right)$ used at least once. No simplification needed. Ignore terms beyond x^3
A1	Two algebraic terms correct in the expansion for their $(1-qx)^{-3}$ .. Must be single fractions, not necessarily in lowest terms. Ignore terms beyond x^3
A1	All four terms correct and in lowest terms. Ignore terms beyond x^3
(c)	
M1	For either their $\frac{1}{8}a=\frac{3}{8}$ or their $\frac{3}{16}a+$ their $\frac{1}{8}b=-\frac{43}{16}$ May be implied by a correct value of *a* or *b*.
A1	$a=3$
A1	$b=-26$ **NB** answers $a=3$ and $b=-26$ scores 3/3
(d)	
M1	Substituting their *a* and their *b* into their $\frac{3a+3b}{16}$
A1	$-\frac{69}{16}$ oe **NB** If $p=2,\ q=\frac{1}{2}$ is used then $a=\frac{3}{16}$ and $b=-\frac{13}{8}$ which substituted into $3a+3b$ gives an answer of $-\frac{69}{16}$ but scores A0

Question number	Scheme	Marks
10 (a)	$\int\left(3x^2-4x-p\right)\,\mathrm{d}x=\frac{3x^3}{3}-\frac{4x^2}{2}-px+c\quad\left[=x^3-2x^2-px+c\right]$	M1A1
	$y=x^3-2x^2-px+c$	
	At $(2,0)$ $\quad 0=8-8-2p+c\Rightarrow c=2p$	M1
	At $(-1,9)$ $\quad 9=-1-2+p+c\Rightarrow c=12-p$	M1
	$\Rightarrow p=4,c=8\Rightarrow y=x^3-2x^2-4x+8\,*$	A1A1cso [6]
(b)	$x^3-2x^2-4x+8=8-4x\Rightarrow x^3-2x^2=0\Rightarrow x^2(x-2)=0$	M1
	$x=0,x=2$	A1
	Area $=\int_0^2(8-4x)\,\mathrm{d}x-\int_0^2\left(x^3-2x^2-4x+8\right)\,\mathrm{d}x$	M1
	Area $=\int_0^2(8-4x)\,\mathrm{d}x-\int_0^2\left(x^3-2x^2-4x+8\right)\,\mathrm{d}x=\int_0^2\left(-x^3+2x^2\right)\,\mathrm{d}x$	
	Area $=\left[-\frac{x^4}{4}+\frac{2x^3}{3}\right]_0^2=\left(-4+\frac{16}{3}\right)-(0)=\frac{4}{3}$	M1M1A1 [6]
		Total 12 marks

(a)	
M1	Attempts to integrate
A1	Correct integration including + c
M1	Substitution of (2, 0) (Does not have to be simplified)
M1	Substitution of (–1, 9 (Does not have to be simplified)
A1	$p=4,c=8$
A1 cso	Obtains the **given** answer with no errors in the working
(b)	
M1	Equating C and l
A1	$x=0$ and $x=2$ **NB** If correct limits are seen then M1A1 is awarded
M1	Use of $\int_a^b\left(\mathrm{f}(x)-\mathrm{g}(x)\right)\mathrm{d}x$ or $\int_a^b\mathrm{f}(x)\mathrm{d}x-\int_a^b\mathrm{g}(x)\mathrm{d}x$ or $\int_a^b\mathrm{f}(x)\mathrm{d}x-\frac{1}{2}\times2\times8$ Ignore limits (These can be either way round)
M1	Attempt the integration. Limits not needed.
M1	Substitute the correct limits. $\left(\text{May be implied by }\pm\frac{4}{3}\right)$
A1	$\frac{4}{3}$ **NB** If no integration is seen then M0M0A0 is awarded for the last 3 marks for an answer of $\frac{4}{3}$

Question number	Scheme	Marks
11 (a)	(i) $y=3$ (ii) $x=-1$	B1B1 [2]
(b) (i)	$0=\dfrac{3x-2}{x+1}\Rightarrow 3x=2\Rightarrow x=\dfrac{2}{3}$	B1
(ii)	$y=\dfrac{3\times 0-2}{0+1}=-2$	B1 [2]
(c)		B1 (shape and position) B1 (asymptotes) B1 (intersections) [3]
(d)	$mx+4=\dfrac{3x-2}{x+1}\Rightarrow mx^2+x(m+1)+6=0$ $b^2-4ac<0\Rightarrow(m+1)^2-4\times m\times 6<0\Rightarrow m^2-22m+1<0$ $(m-11)^2-120=0\Rightarrow m=11\pm\sqrt{120}$ Defines region $11-2\sqrt{30}<m<11+2\sqrt{30}$	M1 M1 M1A1 M1 depM1A1 [7]
		Total 14 marks

(a) (i)	
B1	$y = 3$
(a) (ii)	
B1	$x = -1$
(b) (i)	
B1	$x = \frac{2}{3}$
(b) (ii)	
B1	$y = -2$
(c)	
B1	Two curves in the correct quadrant (Condone poor shape as long as intention is clear)
B1	Two asymptotes drawn and correctly labelled (Allow –1 and 3 correctly labelled on axes to count as labelled). There must be at least one part of the curve drawn
B1	Intersections labelled
(d)	
M1	Equates C and l (Condone use of $\neq$ and <)
M1	Forms a 3TQ (Condone the use of $\neq$ and <) (allow 1 error in the expansion of correct brackets or 1 error in simplifying)
M1	Use of $b^2 - 4ac < 0$ to form a 3TQ
A1	Simplify to $m^2 - 22m + 1 < 0$
M1	Solving the 3TQ If the quadratic is incorrect then the method for solving must be shown
depM1	Simplifying to the form $a \pm b\sqrt{2}$ Dependant on previous M1
A1	$11 - 2\sqrt{30} < m < 11 + 2\sqrt{30}$ (Allow $a = 11$, $b = 30$)

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